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SOFA Vector/Matrix Library
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## PREFACE

The routines described here comprise the SOFA vector/matrix library. Their general appearance and coding style conforms to conventions agreed by the SOFA Review Board, and their functions, names and algorithms have been ratified by the Board. Procedures for soliciting and agreeing additions to the library are still evolving.

## PROGRAMMING LANGUAGES

The SOFA routines are available in two programming languages at present: Fortran 77 and ANSI C.

There is a one-to-one relationship between the two language versions. The naming convention is such that a SOFA routine referred to generically as "EXAMPL" exists as a Fortran subprogram iau_EXAMPL and a C function iauExampl. The calls for the two versions are very similar, with the same arguments in the same order. In a few cases, the C equivalent of a Fortran SUBROUTINE subprogram uses a return value rather than an argument.

## GENERAL PRINCIPLES

The library consists mostly of routines which operate on ordinary Cartesian vectors ( $x, y, z$ ) and $3 x 3$ rotation matrices. However, there is also support for vectors which represent velocity as well as position and vectors which represent rotation instead of position. The vectors which represent both position and velocity may be considered still to have dimensions (3), but to comprise elements each of which is two numbers, representing the value itself and the time derivative. Thus:

* "Position" or "p" vectors (or just plain 3-vectors) have dimension (3) in Fortran and [3] in C.
* "Position/velocity" or "pv" vectors have dimensions $(3,2)$ in Fortran and [2][3] in C.
* "Rotation" or "r" matrices have dimensions (3,3) in Fortran and [3] [3] in C. When used for rotation, they are "orthogonal"; the inverse of such a matrix is equal to the transpose. Most of the routines in this library do not assume that r-matrices are necessarily orthogonal and in fact work on any $3 x 3$ matrix.
* "Rotation" or "r" vectors have dimensions (3) in Fortran and [3] in C. Such vectors are a combination of the Euler axis and angle and are convertible to and from r-matrices. The direction is the axis of rotation and the magnitude is the angle of rotation, in radians. Because the amount of rotation can be scaled up and down simply by multiplying the vector by a scalar, r-vectors are useful for representing spins about an axis which is fixed.
* The above rules mean that in terms of memory address, the three velocity components of a pv-vector follow the three position components. Application code is permitted to exploit this and all other knowledge of the internal layouts: that $x, y$ and $z$ appear in that order and are in a right-handed Cartesian coordinate system etc. For example, the cp function (copy a p-vector) can be used to copy the velocity component of a pv-vector (indeed, this is how the CPV routine is coded).
* The routines provided do not completely fill the range of operations that link all the various vector and matrix options, but are confined to functions that are required by other parts of the SOFA software or which are likely to prove useful.

In addition to the vector/matrix routines, the library contains some routines related to spherical angles, including conversions to and from sexagesimal format.

Using the library requires knowledge of vector/matrix methods, spherical trigonometry, and methods of attitude representation. These topics are covered in many textbooks, including "Spacecraft Attitude Determination and Control", James R. Wertz (ed.), Astrophysics and Space Science Library, Vol. 73, D. Reidel Publishing Company, 1986.

OPERATIONS INVOLVING P-VECTORS AND R-MATRICES
Initialize
ZP zero p-vector
ZR initialize r-matrix to null
IR initialize r-matrix to identity
Copy/extend/extract

| CP | copy |
| :--- | :--- |
| CR -vector |  |
| copy $r$-matrix |  |

Build rotations
RX rotate $r$-matrix about $x$
RY rotate r-matrix about $y$
RZ rotate $r$-matrix about $z$

Spherical/Cartesian conversions
S2C spherical to unit vector
C2S unit vector to spherical
S2P spherical to p-vector
P2S p-vector to spherical
Operations on vectors

```
PPP p-vector plus p-vector
PMP p-vector minus p-vector
PPSP p-vector plus scaled p-vector
PDP inner (=scalar=dot) product of two p-vectors
PXP outer (=vector=cross) product of two p-vectors
PM modulus of p-vector
PN normalize p-vector returning modulus
SXP multiply p-vector by scalar
```

Operations on matrices

| RXR | r-matrix multiply |
| :--- | :--- |
| TR | transpose r-matrix |

Matrix-vector products

```
RXP product of r-matrix and p-vector
TRXP product of transpose of r-matrix and p-vector
```

Separation and position-angle

| SEPP | angular separation from p-vectors |
| :--- | :--- |
| SEPS | angular separation from spherical coordinates |
| PAP | position-angle from p-vectors |

PAS position-angle from spherical coordinates

Rotation vectors
RV2M r-vector to r-matrix

RM2V r-matrix to r-vector

```
Initialize
```

    ZPV zero pv-vector
    Copy/extend/extract

```
CPV Copy pv-vector
P2PV append zero velocity to p-vector
PV2P discard velocity component of pv-vector
```

Spherical/Cartesian conversions
S2PV spherical to pv-vector
PV2S pv-vector to spherical
Operations on vectors

| PVPPV | pv-vector plus pv-vector |
| :--- | :--- |
| PVMPV | pv-vector minus pv-vector |
| PVDPV | inner (=scalar=dot) product of two pv-vectors |
| PVXPV | outer (=vector=cross) product of two pv-vectors |
| PVM | modulus of pv-vector |
| SXPV | multiply pv-vector by scalar |
| S2XPV | multiply pv-vector by two scalars |
| PVU | update pv-vector |
| PVUP | update pv-vector discarding velocity |
|  |  |

```
RXPV product of r-matrix and pv-vector
TRXPV product of transpose of r-matrix and pv-vector
```


## OPERATIONS ON ANGLES

| ANP | normalize radians to range 0 to 2pi |
| :--- | :--- |
| ANPM | normalize radians to range - pi to +pi |
| A2TF | decompose radians into hms |
| A2AF | decompose radians into d " |
| D2TF | decompose days into hms |

## CALLS: FORTRAN VERSION



```
CALL iau_PXP ( A, B, AXB )
CALL iau_RM2V ( \(R, P\) )
CALL iau_RV2M ( \(\mathrm{P}, \mathrm{R}\) )
CALL iau_RX ( PHI, R )
CALL iau_RXP ( \(\mathrm{R}, \mathrm{P}, \mathrm{RP}\) )
CALL iau_RXPV ( R, PV, RPV )
CALL iau_RXR ( A, B, ATB )
CALL iau_RY ( THETA, R )
CALL iau_RZ ( PSI, R )
CALL iau_S2C ( THETA, PHI, C )
CALL iau_S2P ( THETA, PHI, R, P )
CALL iau_S2PV ( THETA, PHI, R, TD, PD, RD, PV )
CALL iau_S2XPV ( S1, S2, PV )
CALL iau_SEPP ( A, B, S )
CALL iau_SEPS ( AL, AP, BL, BP, S )
CALL iau_SXP ( \(S, P, S P\) )
CALL iau_SXPV ( \(S, P V, S P V)\)
CALL iau_TR ( R, RT )
CALL iau_TRXP ( R, P, TRP )
CALL iau_TRXPV ( \(\mathrm{R}, \mathrm{PV}, \mathrm{TRPV}\) )
CALL iau_ZP ( P )
CALL iau_ZPV ( PV )
CALL iau_ZR ( R )
```

CALLS: C VERSION

```
    iauA2af ( ndp, angle, &sign, idmsf );
    iauA2tf ( ndp, angle, &sign, ihmsf );
d = iauAnp ( a );
d = iauAnpm ( a );
    iauC2s ( p, &theta, &phi );
    iauCp ( p, c );
    iauCpv ( pv, c );
    iauCr ( r, c );
    iauD2tf ( ndp, days, &sign, ihmsf );
    iauIr ( r );
    iauP2pv ( p, pv );
    iauP2s ( p, &theta, &phi, &r );
d = iauPap ( a, b );
d = iauPas ( al, ap, bl, bp );
d = iauPdp ( a, b );
d = iauPm ( p );
    iauPmp ( a, b, amb );
    iauPn ( p, &r, u );
    iauPpp ( a, b, apb );
    iauPpsp ( a, s, b, apsb );
    iauPv2p ( pv, p );
    iauPv2s ( pv, &theta, &phi, &r, &td, &pd, &rd );
    iauPvdpv ( a, b, adb );
    iauPvm ( pv, &r, &s );
    iauPvmpv ( a, b, amb );
    iauPvppv ( a, b, apb );
    iauPvu ( dt, pv, upv );
    iauPvup ( dt, pv, p );
    iauPvxpv ( a, b, axb );
    iauPxp ( a, b, axb );
    iauRm2v ( r, p );
    iauRv2m ( p, r );
    iauRx ( phi, r );
    iauRxp ( r, p, rp );
    iauRxpv ( r, pv, rpv );
    iauRxr ( a, b, atb );
    iauRy ( theta, r );
    iauRz ( psi, r );
    iauS2c ( theta, phi, c );
    iauS2p ( theta, phi, r, p );
    iauS2pv ( theta, phi, r, td, pd, rd, pV );
    iauS2xpv ( s1, s2, pv );
    d = iauSepp ( a, b );
    d = iauSeps ( al, ap, bl, bp );
    iauSxp ( s, p, sp );
    iauSxpv ( s, pv, spv );
```

iauTr ( r, rt );
iauTrxp ( r, p, trp );
iauTrxpv ( r, pv, trpv );
iauZp ( p );
iauZpv ( pv );
iauZr ( r );

